LITERATURE CITED

1. N. E. Kochin and L. G. Loitsyanskii, "On an approximate method to compute the laminar boundary layer," Dokl. Akad. Nauk SSSR, 36, No. 9, 278-284 (1942).
2. O. G. Martynenko and Yu. A. Sokovichin, Freely Convective Heat Transfer on a Vertical Surface (Boundary Conditions of the Second Kind) [in Russian], Nauka i Tekhnika, Minsk (1977).
3. E. M. Sparrow and J. L. Gregg, "Similar solutions for free convection from a nonisothermal vertical plate," Trans. ASME, 80, No. 2, 379-386 (1958).
4. S. B. Sutton, "A study of free convection from a vertical plate with sinusoidal temperature distribution," AIAA Paper No. 71-988 (1971).
5. T. T. Kao, "Locally nonsimilar solution for laminar free convection adjacent to a vertical wall," Trans. ASME, J. Heat Transfer, 98C, No. 2, 321-322 (1976).
6. T. T. Kao, G. A. Domoto, and H. G. Elrod, Jr., "Free convection along a nonisothermal vertical flat plate," Trans. ASME, J. Heat Transfer, 99C, No. 1, 72-78 (1977).
7. T. Y. Na, Numerical solution of natural convection flow past a nonisothermal vertical flat plate," Appl. Sci. Res., 33, Nos. 5/6, 519-543 (1978).

MOTION OF A THERMAL WAVE FRONT IN A NONLINEAR MEDIUM WITH ABSORPTION

I. S. Granik and L. K. Martinson

UDC 536.24

We study the evolution of a thermal perturbation in a nonlinear medium whose thermal conductivity depends on the temperature and the temperature gradient according to a power law.

We consider an incompressible medium whose thermal conductivity depends on the temperature and temperature gradient according to the power law

$$k = k_0 u^\sigma |\mathrm{grad}\, u|^\alpha, \quad \sigma,\ \alpha = \mathrm{const} > 0.$$

Such a model of a medium, withgeneralization of the models used in the theory of nonlinear heat conduction [1], wasvalidated in [2] from the point of view of kinetic theory as a model of a medium with a finite relaxation time.

As follows from the results of [3], thermal perturbations in such a medium, unlike a medium with constant thermal conductivity, can be generalized to the form of thermal waves with finite velocity of displacement of the fronts. Below, we study the features of the motion of thermal wave fronts from an instantaneous point source of heat in the presence in a given nonlinear medium of volume absorption of thermal energy, the intensity of which depends on temperature according to a power law. Such absorption of thermal energy can be caused by processes of ionization and radiation in a high-temperature medium [1, 4].

The corresponding process of propagation of heat is described by a Cauchy problem for the quasilinear parabolic equation

$$\frac{\partial u}{\partial t} = \frac{a^2}{x^{s-1}} \frac{\partial}{\partial x}\left(x^{s-1} u^\sigma \left|\frac{\partial u}{\partial x}\right|^\alpha \frac{\partial u}{\partial x}\right) - \Pi u^\nu, \quad t > 0, \quad x > 0,$$

$$u(0,\ x) = Q_0 \delta(x^s). \tag{1}$$

Here Q_0 is the energy of a point thermal source at the initial time; $\Pi = \mathrm{const} > 0$ is the coefficient of absorption, and s = 1, 2, and 3 for the cases of plane, axial, and central symmetries of the problem, respectively. Below, without loss of generality, we assume $a^2 = 1$, since by the choice of the time scale we can always reduce (1) to such form.

For certain values of the exponent ν in the lowest term of the equation we can find exact analytical solutions of the problem (1). An analysis of these solutions shows that

N. É. Bauman Moscow Higher Technical School. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 39, No. 4, pp. 728-731, October, 1980. Original article submitted July 16, 1979.

the volume absorption of heat changes the nature of the motion of the thermal wave front — not only quantitatively but also qualitatively. We investigate certain of these solutions.

1. We consider the case when $\nu = 1$. Then, using the substitution

$$u(t, x) = v(t, x)\exp(-\Pi t),$$
$$\tau(t) = \tau_m[1 - \exp(-t/\tau_m)], \quad \tau_m = [\Pi(\alpha + \sigma)]^{-1} \tag{2}$$

we transform (1) to a Cauchy problem in the half-strip $G = \{(\tau, x): 0 \leq \tau < \tau_m, x \geq 0\}$ for the function $v(\tau(t), x) \equiv v(t, x)$

$$\frac{\partial v}{\partial \tau} = \frac{1}{x^{s-1}}\frac{\partial}{\partial x}\left(x^{s-1}v^{\sigma}\left|\frac{\partial v}{\partial x}\right|^{\alpha}\frac{\partial v}{\partial x}\right),$$
$$v(0, x) = Q_0\delta(x^s). \tag{3}$$

The solution of this problem has the form [3]

$$v(\tau, x) = \begin{cases} A\left[\dfrac{x_+^{\alpha+2}(\tau)}{\tau}\right]^{1/\alpha+\sigma}\left[1 - \left(\dfrac{x}{x_+(\tau)}\right)^{\alpha+2/\alpha+1}\right]^{\alpha+1/\alpha+\sigma}, & x < x_+(\tau), \\ 0, & x \geqslant x_+(\tau), \end{cases} \tag{4}$$

where

$$A = \left[r\left(\frac{\alpha+\sigma}{\alpha+2}\right)^{\alpha+1}\right]^{1/\alpha+\sigma},$$
$$r = \left[(\alpha+\sigma)\left(s + \frac{\alpha+2}{\alpha+\sigma}\right)\right]^{-1}, \quad x_+(\tau) = C\tau^r, \tag{5}$$
$$C = Q_0^{r(\alpha+\sigma)}\left\{L(s)A\frac{\alpha+1}{\alpha+2}B\left[\frac{s(\alpha+1)}{\alpha+2}, \quad 1 + \frac{\alpha+1}{\alpha+\sigma}\right]\right\}^{-r(\alpha+\sigma)},$$
$$L(s) = \begin{cases} 2 & \text{for } s = 1, \\ 2\pi & \text{for } s = 2, \\ 4\pi & \text{for } s = 3. \end{cases}$$

From (4) with account of (2) it follows that the thermal perturbation from the source is propagated in the form of a thermal wave, where the position of its front at any time is defined by the relation

$$x = x_+(t) = C\tau_m^r[1 - \exp(-t/\tau_m)]^r. \tag{6}$$

As can be seen from (6), in a medium with volume absorption of heat of indicated form ($\nu = 1$), we observe the effect of spatial localization of a thermal perturbation [5], when even after an infinite time interval, the thermal wave front penetrates into the medium to only a finite depth, i.e., $x_+(t) \leq x_m < +\infty$ for any $t \in [0, +\infty)$.

Actually, from (6) it follows that for $t \to +\infty$

$$x_+(t) \to x_m = C\tau_m^r = C[\Pi(\alpha+\sigma)]^{-r}. \tag{7}$$

From (7) we have

$$x_m \sim (Q_0^{\alpha+\sigma}/\Pi)^r.$$

The depth of penetration of the thermal wave front into a nonlinear medium with absorption decreases with decreasing initial thermal energy of the point source and with increasing value of the absorption coefficient.

2. Let the exponent of the lowest term in (1) be $\nu = (1 - \sigma)/(1 + \alpha)$ ($\sigma<1$, $\nu<1$). The exact solution of (1) in this case can be found in the explicit analytic form

$$u(t, x) = \begin{cases} At^{-1/\alpha+\sigma}[x_+^p(t) - x^p]^{\alpha+1/\alpha+\sigma}, & x < x_+(t), \\ 0, & x \geqslant x_+(t), \end{cases} \tag{8}$$

where $p = (\alpha + 2)/(\alpha + 1)$, and the function $x = x_+(t)$, determining the position of the thermal wave front at any time, has the form

$$x = x_+(t) = Ct\,[t^{p(r-1)} - R]^{1/p},$$
$$R = \Pi\,[(1-r)\,r^{1/\alpha+1}C^p]^{-1}. \quad (9)$$

In Eqs. (8) and (9), the values of the constants A and C, and also of the parameter r, are defined by Eqs. (5).

From (9) it follows that, at first, in the medium there propagates a heating wave, when $x_+(t) > 0$. During the motion, the velocity of the thermal wave front decreases, and for $t = T_0$, where

$$T_0 = (r/R)^{1/p(1-r)},$$

the front stops, having penetrated into the medium to a depth

$$x_m = x_+(T_0) = CT_0\left(R\,\frac{1-r}{r}\right)^{1/p}.$$

Thus, in this problem, absorption also leads to the spatial localization of thermal perturbations.

For $t > T_0$, the heating wave alternates with a cooling wave ($\dot{x}_+(t) < 0$ for $t > T_0$), the thermal wave front changes the direction of its motion, and the dimension of the region of the thermal perturbation begins to decrease with time.

At time $t = T_m = R^{-1/p(1-r)}$, the perturbation region contracts to a point and for $t > T_m$, $u(t, x) = 0$ for all $x \geq 0$. In other words, in the problem being considered, the thermal perturbation of the instantaneous point source exists in a nonlinear medium with volume absorption of thermal energy only for a finite time T_m. It can easily be seen that $T_m \to +\infty$ as $\Pi \to 0$, i.e., a nonlinear effect of a finite time of existence of the thermal perturbation is due to the effect of volume absorption of heat.

We can show that a thermal perturbation in the considered nonlinear medium with absorption exists for a finite time if $\nu < 1$. This can be verified, considering in such a medium the initial thermal perturbation in the form of an isothermal background with temperature $u_0 > 0$. Because of heat absorption, the temperature of the medium will decrease, where

$$u(t) = \begin{cases} [\Pi\,(1-\nu)(T_* - t)]^{1/1-\nu}, & t < T_*, \\ 0, & t \geqslant T_*, \end{cases} \quad (10)$$

and

$$T_* = \frac{u_0^{1-\nu}}{\Pi\,(1-\nu)}.$$

Therefore, for $\nu < 1$, any initial perturbation with maximum temperature $u^{max}(0, x) \leq u_0$ will exist in a medium with absorption for a finite time $T_m \leq T_*$.

NOTATION

u, temperature; k, coefficient of thermal conductivity; t, time; x, spatial variable; x_+, a point on the thermal wave front; a^2, generalized coefficient of thermal diffusivity; σ, α, ν, and s, parameters of the process; $\delta(x^s)$, Dirac delta-function; $B[\xi, \eta]$, a beta function; $v(\tau, x)$, $\tau(t)$, auxiliary functions; A, C, T_0, T_m, T_*, R, r, p, and τ_m, constants and parameters.

LITERATURE CITED

1. Ya. B. Zel'dovich and Yu. P. Raizer, Physics of Shock Waves and High-Temperature Hydrodynamic Phenomena, Academic Press (1966).
2. B. M. Berkovsky and V. G. Bashtovoi, "The finite velocity of heat propagation from the viewpoint of the kinetic theory," Int. J. Heat Mass Transfer, 20, No. 6, 621-626 (1977).
3. G. I. Barenblatt, "Self-similar motions of a compressible liquid in a porous medium," Prikl. Mat. Mekh., 16, No. 6, 679-698 (1952).
4. M. S. Chu, "Thermonuclear reaction waves at high densities," Phys. Fluids, 15, No. 3, 413-422 (1972).
5. L. K. Martinson and K. B. Pavlov, "Spatial localization of thermal perturbations in the theory of nonlinear heat conduction," Zh. Vychisl. Mat. Mat. Fiz., 12, No. 4, 1048-1053 (1972).